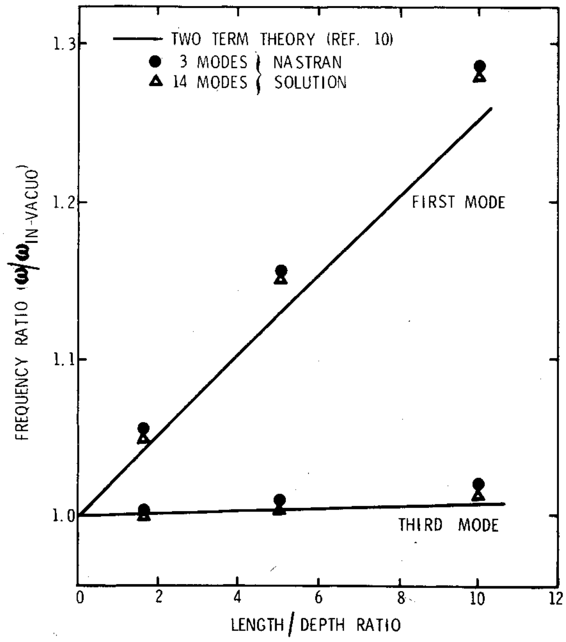

Fig. 2 Cavity effect on panel natural frequencies.

The distribution of modal energies is often of interest in interpreting synthesis results, especially for more complicated structural geometries. For this purpose, the substructure modes are mass normalized prior to inclusion in the synthesis. Then each element of an eigenvector of the synthesis is proportional to the squre root of the kinetic energy in the associated substructure mode. The distribution of kinetic energy in two modes of the panel-cavity system is given in Table 1. The interaction between panel and cavity is greater for a shallower cavity (larger length/depth ratio), as indicated by the increase in energy for the cavity (Helmholtz) mode shown in the last column of Table 1. Note also that for a given length/depth ratio the interaction is greater for the first mode than for the third mode, as mentioned in Ref. 10.

Conclusions

The equations of motion for a combined structural-acoustic system have been reduced to the form of a modal synthesis. This formulation can then be used to economically obtain the system eigenvalues and eigenvectors. These results are of interest in themselves or may serve as the basis for a forced solution.

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Variable Thickness Shear Layer Aerodynamics Revisited

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VENTRES,[1] Chi,[2] and Williams et al.,[3] have studied lifting surface aerodynamics in a parallel, shear flow of constant thickness. To improve the accuracy and realism of this model, particularly in its application to boundary-layer flows, Chi[2] has extended Ventres' solution for this model in steady, two-dimensional incompressible flow to allow for variation of the shear (boundary) layer thickness along the lifting surface chord. His solution, though correct, is unnecessarily elaborate. It is the purpose of the present Note to point out that for shear layers of slowly varying thickness, it is sufficient to replace the constant boundary-layer thickness which appears in Ventres' Kernel function by the varying thickness. The Kernel function in question is that relating pressure on the lifting surface to its downwash (normal velocity on its surface).

Chi[2] has shown for a slowly varying shear layer thickness, $\delta(x)$, that

$$w(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K^*[\alpha,\delta(x)]p_w^*[\alpha,\delta(x)]e^{i\alpha x}\mathrm{d}\alpha \tag{1}$$

Here x = coordinate along airfoil chord, w = downwash, K^* = Fourier transform of Ventre's Kernel *for constant* δ with δ now taken as a function of x, and p_w^* = Fourier component of wall pressure. The (physical) wall pressure is given by

$$p_w[x,\delta(x)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} p_w^*[\alpha,\delta(x)]e^{i\alpha x}\mathrm{d}\alpha \tag{2}$$

From Eq. (2), it seems useful to define

$$p_w[x',\delta(x)] \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} p_w^*[\alpha,\delta(x)]e^{i\alpha x'}\mathrm{d}\alpha \tag{3}$$

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From Eq. (3) and the usual theory of Fourier Transform pairs,

$$p_w^*[\alpha,\delta(x)] = \int_{-\infty}^{\infty} p_w[x',\delta(x)]e^{-i\alpha x'}\mathrm{d}x' \tag{4}$$

Using Eq. (4) in Eq. (1),

$$w(x) = \int K[x-x',\delta(x)]p_w[x',\delta(x)]\mathrm{d}x' \quad \text{over airfoil chord} \tag{5}$$

where $p_w[x',\delta(x)]$ is given by Eq. (3) and

$$K \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} K^*[\alpha,\delta(x)]e^{i\alpha(x-x')}\mathrm{d}\alpha \tag{6}$$

Aside: from the theory of Fourier Transform pairs

$$K^*[\alpha,\delta(x)] \neq \int_{-\infty}^{\infty} K[x-x',\delta(x)]e^{-i\alpha(x-x')}\mathrm{d}(x-x')$$

and this led Chi[2] to adopt a somewhat indirect and more approximate solution procedure. However as we show below the above result does not prevent the solution of (1) or (5) in the usual way.[1-3]

Solution for $p_w[x,\delta(x)]$

This can be done in two (equivalent) ways, using Eq. (1) or Eq. (5). First consider Eq. (1) from which the result is more transparent.

If we expand p_w,

$$p_w[x',\delta(x)] = \sum_{m=1}^{M} b_m[\delta(x)]\psi_m(x') \tag{7}$$

where the ψ_m are chosen functions (satisfying the Kutta condition at the trailing edge), then from Eq. (4)

$$p_w^*[\alpha,\delta(x)] = \sum_{m=1}^{M} b_m[\delta(x)]\psi_m^*(\alpha) \tag{8}$$

Substituting Eq. (8) into Eq. (1) and employing the usual collection procedure by evaluating the result at $x=x_i$, $i=1,\ldots,M$ where M is the max of m, one has

$$w(x_i) = \sum_{m=1}^{M} b_m[\delta(x_i)]\frac{1}{2\pi}\times\int_{-\infty}^{\infty} K^*[\alpha,\delta(x_i)]\psi_m^*(\alpha)e^{i\alpha x_i}\mathrm{d}\alpha \tag{9}$$

The b_m [and hence p_w from Eq. (7)] are determined by solving the set of linear algebraic equations, Eq. (9). Note that for constant δ, the b_m depend parametrically on δ also. Indeed, b_m [and in particular its dependence on $\delta(x)$] is determined by $w(x)$ and $K[x-x',\delta(x)]$ for constant or variable $\delta(x)$.

An alternative, equivalent approach is to use Eq. (7) in Eq. (5) and obtain

$$w(x_i) = \sum_{m=1}^{M} b_m[\delta(x_i)]\int K[x_i-x',\delta(x_i)]\psi_m(x')\mathrm{d}x' \tag{10}$$

from which the b_m can again be determined.

Once the b_m are known, then from Eq. (7),

$$p_w[x',\delta(x)] = \sum_{m=1}^{M} b_m[\delta(x)]\psi_m(x')$$

and

$$p_w[x,\delta(x)] = \sum_{m=1}^{M} b_m[\delta(x)]\psi_m(x) \tag{11}$$

In Figs. 1 and 2, results are presented from the present approach and compared to the previous results.[2] For $\delta/c<.1$, typical of most applications, the present results are in good agreement with those obtained in Ref. 2. For $\delta/c>.1$, presumably the present results are more accurate and were obtained with substantially less effort.

As the attentive reader of Refs. 1-3 and the present Note will have concluded, the above result will still hold formally for three-dimensional, compressible, and unsteady flows. Of

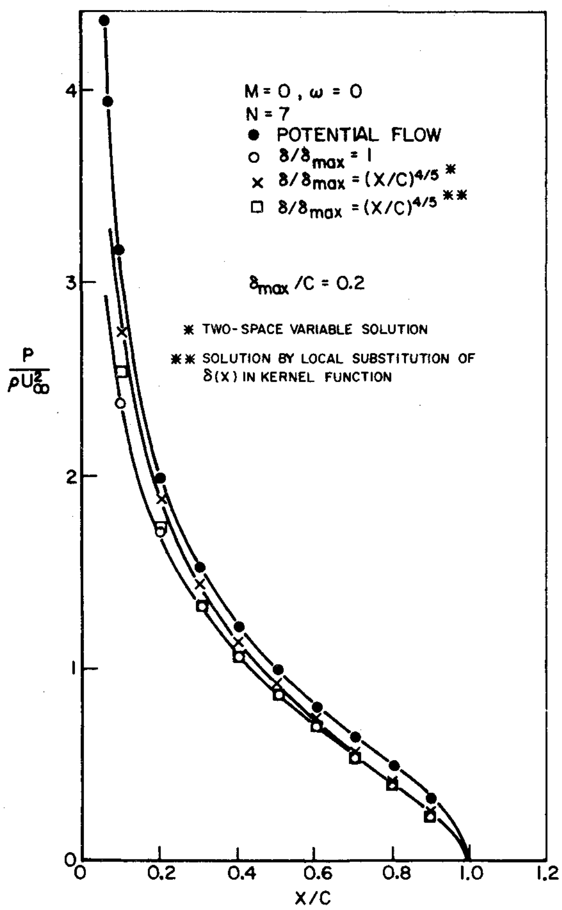

Fig. 1 Pressure distribution along chord.

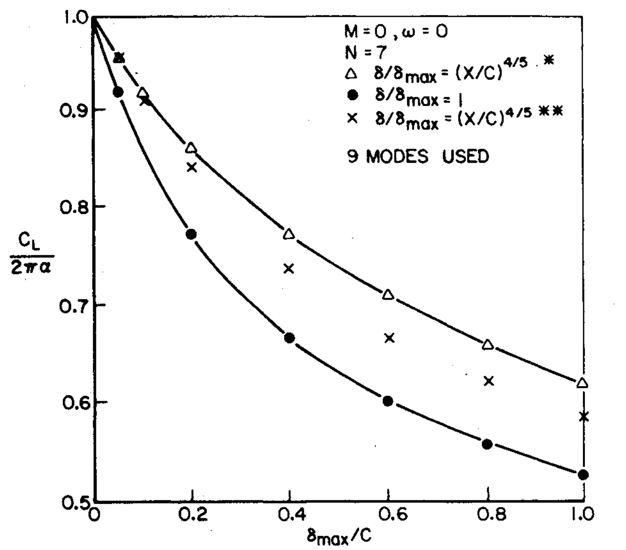

Fig. 2 Lift coefficient vs δ_{max}/C for both uniform and variable boundary-layer thickness of the case $N=7$. *Two-space variable solution. **Solution by local substitution of $\delta(x)$ in kernel function.

course, the range of accuracy of the basic assumption of a slowly varying shear layer thickness in any of these cases remains to be determined. It is noted that if the shear layer thickness varies rapidly, i.e. $d\delta/dx$ is not sufficiently small, then the shear layer model itself becomes inapplicable.[4] Fortunately K becomes independent of δ^1 if $(x-x')/\delta(x) \gtrsim 1$ and δ itself does not vary significantly over a distance δ along the airfoil chord if $\delta/c \ll 1$.

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Three-Dimensional Compressible Stagnation Point Boundary Layers with Large Rates of Injection

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Introduction

THE study of the steady laminar compressible boundary layers with large mass injection rates at a general three-dimensional stagnation point is of considerable importance in high-speed flight of spacecraft entering or re-entering planetary atmospheres. It is known that the large rates of injection in the boundary layer significantly alter the flow features from those usually expected. In particular, it is known that the boundary layer with large injection rates involves a structure consisting of a relatively thick inner layer close to the surface where viscous forces are negligible compared with pressure and inertia forces and of a relatively thin outer layer providing the adjustment of the inner layer to the inviscid external flow. The effects of moderate injection on the steady laminar compressible three-dimensional forward stagnation point boundary-layer flow of a gas with constant $\rho\mu$ flows ($\rho \propto T^{-1}$, $\mu \propto T$, $Pr=0.7$ where ρ,μ,T and Pr are the density, viscosity, temperature, and Prandtl number, respectively) have been studied by Libby[1] and with variable $\rho\mu$ flows ($\rho \propto T^{-1}$, $\mu \propto T^{\omega}$, $Pr=0.7$ where ω is the index of the power-law variation of viscosity) by Wortman and Mills[2] and Vimala and Nath.[3] Recently, for constant $\rho\mu$ flows, Libby[4] obtained an exact and an approximate solution of the above problem with large injection rates. The exact solution was obtained using quasilinearization techniques,[1] and the approximate solution using matched asymptotic expansions.[4] However, for variable gas properties (variable $\rho\mu$ flows), the solution with large injection rates has not been obtained before.

The aim of the present analysis is to obtain an exact solution of the preceding problem with variable gas properties and large injection rates. The governing equations were solved numerically using quasilinearization technique.[1] The results clearly indicate the inadequacy of solutions obtained under the simplifying assumption that $\rho\mu$=constant across the boundary layer especially for low-wall temperatures or large rates of injection.

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Governing Equations

The governing equations in dimensionless form for the steady laminar compressible boundary-layer flow of a gas with variable properties in the neighborhood of a three-dimensional forward stagnation point with mass injection under similarity assumptions are[1-3]

$$f''' + (\omega-1)g^{-1}g'f'' + [(f+cF)f'' + g - f'^2]g^{1-\omega} = 0 \quad (1)$$

$$F''' + (\omega-1)g^{-1}g'F'' + [(f+cF)F'' + c(g-F'^2)]g^{1-\omega} = 0 \quad (2)$$

$$g'' + (\omega-1)g^{-1}g'^2 + Pr(f+cF)g'g^{1-\omega} = 0 \quad (3)$$

The boundary conditions are

$$f(0) = f_w,\ f'(0) = F(0) = F'(0) = 0,\ g(0) = g_w \quad (4a)$$

$$f'(\infty) = F'(\infty) = g(\infty) = 1 \quad (4b)$$

Here f and F are the dimensionless stream functions; g is the dimensionless enthalpy; g_w and $f_w = -(\rho w)_w/(\rho_e\mu_e a)^{1/2}$ are the dimensionless enthalpy and injection parameters at the wall, respectively (other symbols are given in Ref. 1). It may be noted that $\omega=0.5$ corresponds to the conditions encountered in hypersonic flight, $\omega=0.7$ corresponds to low-temperature flows, and $\omega=1$ represents the constant density-viscosity product simplification.[5] It is to be mentioned that most shapes of practical interest[1,4] range from sphere $(c=1)$ to cylinder $(c=0)$.

The skin-friction coefficients C_{f_x} and C_{f_y} along the x and y directions and the heat-transfer coefficient in terms of the Stanton number St are given by[1,3]

$$C_{f_x} = 2(Re_x)^{-1/2}f_I''(0),\ C_{f_y} = 2(R_{ex})^{-1/2}(v_e/u_e)F_I''(0) \quad (5)$$

$$St = (Re_x)^{-1/2}G'(0),\ f_I''(0) = g_w^{\omega-1}f''(0),$$
$$F_I''(0) = g_w^{\omega-1}F''(0) \quad (6)$$

$$G'(0) = Pr^{-1}g_w^{\omega-1}g'(0)/(1-g_w),\ Re_x = u_e x/\nu_e \quad (7)$$

where $f_I''(0)$ and $F_I''(0)$ are the skin-friction parameters along the x and y directions, respectively; $G'(0)$ and $F_I''(0)$ are the skin-friction parameters along the x and y directions, respectively; $G(0)$ is the heat-transfer parameter, and Re_x is the local Reynolds number. It may be noted that for $\omega=1$, $g_w^{\omega-1}=1(g_w>0)$ and $g_w=0.2$ and $\omega=0.5$ and $0.7, g_w^{\omega-1} = 2.2361$ and 1.6207, respectively. It is clear that $g_w^{\omega-1}$ is an important parameter in predicting skin friction and heat transfer especially at low-wall temperatures. Therefore, the results obtained under the assumptions that $\rho\mu$=constant $(\omega=1)$ across the boundary layer is of limited engineering value because, in effect, it eliminates the parameter $g_w^{\omega-1}$ from the skin friction and heat transfer.

Results and Discussion

The Eqs. (1-3) under conditions (4) have been solved numerically using a quasilinearization technique[1] for various